given a pin-hole model

$$P \xrightarrow{} \boxed{\begin{aligned} P' &= Rp + t \\ P_c &= \begin{bmatrix} P_x'/P_z' \\ P_y'/P_z' \\ 1 \end{bmatrix} \end{aligned}} \xrightarrow{P_c} \boxed{\begin{aligned} u_c' &= u_c(1 + k_1 r_c^2 + k_2 r_c^4) \\ v_c' &= v_c(1 + k_1 r_c^2 + k_2 r_c^4) \end{aligned}} \xrightarrow{u_c' \, v_c'} \boxed{\begin{aligned} u_s &= f_x u_c' + c_x \\ v_s &= f_y v_c' + c_y \end{aligned}} \xrightarrow{} u_s, v_s$$

$P \in \mathbb{R}^3$

R, t $\quad$ k_1, k_2 $\quad$ f_x, f_y, c_x, c_y

$$z = h(x, y)$$

meas. $\quad$ pose $\quad$ landmark

∴ error:

$$e = z - h(T, p)$$

∴ @ lm p_j w/ pose T_i

overall error:

$$\frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{n} \| e_{ij} \|^2$$

$$= \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{n} \| z_{ij} - h(T_i, p_j) \|^2$$

→ we are adjusting the pose T

& lm p

at the same time

→ Bundle Adjustment

∴ in the final problem:

$$x = [T_1, \dots, T_m, p_1, \dots, p_n]^T$$

∴ when $x \leftarrow x + \Delta x$

$$\frac{1}{2} \| f(x + \Delta x) \|^2$$

$$\approx \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{n} \| e_{ij} + F_{ij} \Delta \xi_i + E_{ij} \Delta p_j \|^2$$

Partial Derivative w.r.t. T_i $\quad$ Partial Derivative w.r.t. P_j

we then stack all variables altogether:

$$x_c = [\xi_1, \xi_2, \dots, \xi_m]^T \in \mathbb{R}^{6m}$$

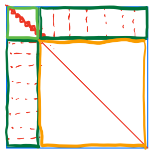
$$x_p = [p_1, p_2, \dots, p_n]^T \in \mathbb{R}^{3n}$$

$$\rightarrow \frac{1}{2} \| f(x + \Delta x) \|^2 = \frac{1}{2} \| e + F \Delta x_c + E \Delta x_p \|^2$$

$$\rightarrow \quad H \Delta x = g$$

- $H = J^T J$ or $J^T J + \lambda I$
- $J = [F \; E]$
- $J^T J = \begin{bmatrix} F^T F & F^T E \\ E^T F & E^T E \end{bmatrix}$

Schur Trick

H = 

$$H = \begin{bmatrix} B & E \\ E^T & C \end{bmatrix}$$

$$\Rightarrow \quad H \Delta x = g \iff \begin{bmatrix} B & E \\ E^T & C \end{bmatrix} \begin{bmatrix} \Delta x_c \\ \Delta x_p \end{bmatrix} = \begin{bmatrix} v \\ w \end{bmatrix}$$