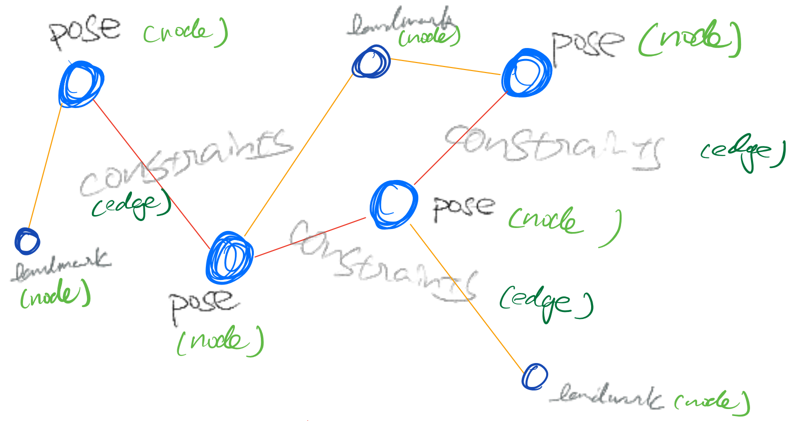
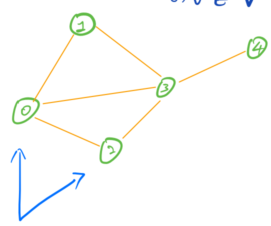


{ edge : dynamic / sensor model
{ node : objective problem

→ solve the over determined problem!

basic graph theory

$G=(V,E)$ E connect u,v ; $u,v \in V$

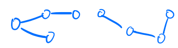


neighbor

$V = \{0,1,2,3,4\}$

$E = \{(0,1), (0,2), (0,3), (1,3), (2,3), (3,4)\}$

1. neighbor(0) = {0, 1, 2}
2. degree (0) = 3
   degree (2) = 2
3. path 4 → 3 → 2 → 0
4. cycle 0 → 1 → 3 → 2 → 0
5. connectivity :- graph is connected if ∃ path between (u,v), $(u,v) \in V$
   - graph is connected when all vertices are connected
   - connected component $V' \subseteq V$

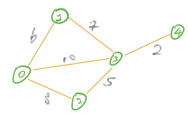


types of graph

1. undirected graph (shown above)
2. directed graph

directed (cyclic) graph   directed Acyclic graph (DAG)

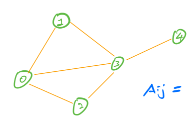
3. weighted graph



4. trees
   1. connected and acyclic
   2. removing edge disconnects graph
   3. adding edge creates a cycle

graph representation

- Adjacency Matrix



$$A_{ij} = \begin{cases} 1 & \text{for edge } (i,j) \\ 0 & \text{otherwise} \end{cases}$$

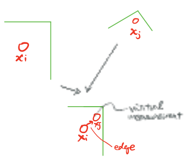
$$\begin{array}{c|ccccc} & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 3 & 1 & 1 & 1 & 0 & 1 \\ 4 & 0 & 0 & 0 & 1 & 0 \end{array}$$

- edge set
  {(0,1), (0,2), (0,3), (1,3), (2,3), (3,4)}
- adjacency list
  0 → [1 2 3]
  1 → [0 3]
  2 → [0 3]
  3 → [0 1 2 4]
  4 → [3]

Edge Creation

- Odometry

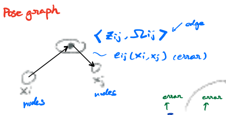
- measurement



edges are then represented as Homogenous coordinates

- odometry
  $x_i^{-1} x_{i+1}$
- observation
  $x_i^{-1} x_j$
  (node i sees node j)

Pose graph



goal: $x^* = \underset{x}{\operatorname{argmin}} \sum_{ij} e_{ij}^T \Omega_{ij} e_{ij}$ (error, error; Ω_{ij}: information matrix)

remark: why do we write it this way

A: covariances? Gaussian? Basically: we're looking at the mean of the Gaussian distribution

assumption: constraints are independent of each other

error function:

$e_{ij}(x_i, x_j) = t2v(z_{ij}^{-1}(x_i^{-1} x_j))$ (z_{ij}: measurement; $x_i^{-1}x_j$: x_j reference w.r.t. x_i)

Gauss-Newton:

- linearization:

$e_{ij}(x + \Delta x) \simeq e_{ij}(x) + J_{ij} \Delta x$

Remark: $J_{ij} = \frac{\partial e_{ij}(x)}{\partial x}$

$e_{ij}(x)$ depends only on x_i & x_j

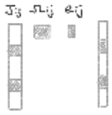
$e_{ij}(x) = e_{ij}(x_i, x_j)$

$J_{ij} = \left[0 \cdots 0 \;\middle|\; \frac{\partial e}{\partial x_i} \;\middle|\; 0 \cdots 0 \;\middle|\; \frac{\partial e}{\partial x_j} \;\middle|\; 0 \cdots \right]$

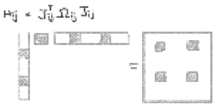
- $b^T = \sum_{ij} b_{ij}^T = \sum_{ij} e_{ij}^T \Omega_{ij} J_{ij}$ (sparse)

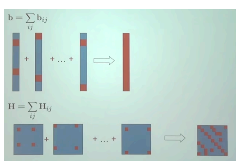
$H = \sum_{ij} H_{ij}^T = \sum_{ij} J_{ij}^T \Omega_{ij} J_{ij}$

$b_{ij} = J_{ij}\ \Omega_{ij}\ e_{ij}$

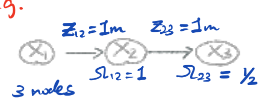


$H_{ij} = J_{ij}^T \Omega_{ij} J_{ij}$





e.g.



3 nodes
2 constraints

- $X_0 = \{x_1, x_2, x_3\} = \{0, 0, 0\}$ initial guess

- $e_{ij} = z_{ij} - (x_j - x_i)$

  $e_{12} = (1 - (0-0)) = 1$

  $e_{23} = (1 - (0-0)) = 1$

- $J_{ij} = \left(\frac{\partial e_{ij}}{\partial x_1} \;\; \frac{\partial e_{ij}}{\partial x_2} \;\; \frac{\partial e_{ij}}{\partial x_3} \right)$

  $J_{12} = \left(\frac{\partial e_{12}}{\partial x_1} \;\; \frac{\partial e_{12}}{\partial x_2} \;\; \frac{\partial e_{12}}{\partial x_3} \right)$

  $= (1 \;\; -1 \;\; 0)$

  $J_{23} = (0 \;\; 1 \;\; -1)$

- $b^T = \sum_{ij} e_{ij}^T \Omega_{ij} J_{ij}$

  $= 1 \cdot 1 \cdot (1, -1, 0)$ (b_1)
  $+ 1 \cdot \frac{1}{2} \cdot (0, 1, -1)$ (b_2)

  $= (1 \;\; -\frac{1}{2} \;\; -\frac{1}{2})$

- $H = \sum_{ij} J_{ij}^T \Omega_{ij} J_{ij}$

  $= \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

  $= \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

- $\Delta x = -H \backslash b$ → ~~error~~ when $\det(H) = 0$

- change the relative constraints to global one

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$