AAE6102 Satellite Communication & Navigation

Ch1 Basics

GPS Architecture

Space segment
- broadcast ranging codes & navigation data (50 bits/second)

Control segment
- track/monitor GPS satellites
- estimate clock & orbit
- small maneuvers to maintain orbit
- small clock corrections
- relocations to compensate SV failure
- distributed globally to ensure coverage
- position surveyed exactly

User segment
- receivers

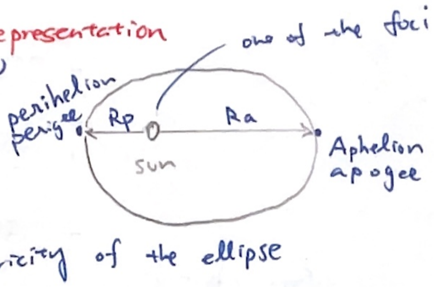
Satellite position representation

Kepler's three law

- **The law of orbit**

$R_a = a(1+e)$

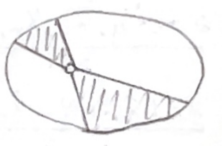
$R_p = a(1-e)$



e= eccentricity of the ellipse

- **The law of areas**

$$\frac{A_0}{t_1-t_0} = \frac{A_1}{t_2-t_1}$$



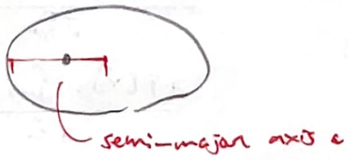
area velocity = const.

- **The law of period**

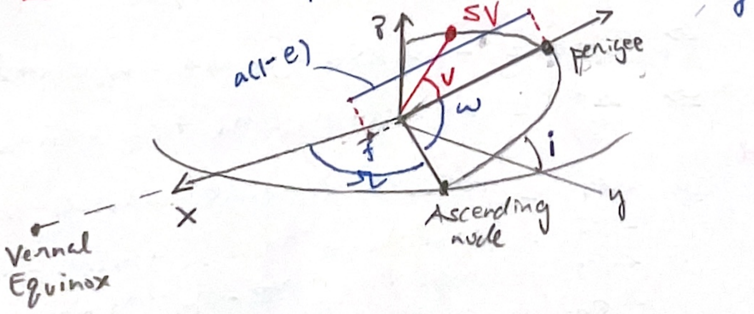
$$\frac{R_0^3}{T_0^2} = \frac{R_1^3}{T_1^2}$$

semi-major axis

orbital period.



Keplerian Elements
- orbit shape : a semimajor axis
  - e eccentricity
- orbit orientation relative to earth : i inclination
  - Ω right ascension of ascending node
  - ω angle of perigee
- Satellite angular position : v true anomaly



△ Coordinate Transformation

Keplerian parameters

↓

Peri-focal coordinates

↓

Earth centered inertial

↓

Earth centered earth-fixed (World Geodetic Survey WGS-84)

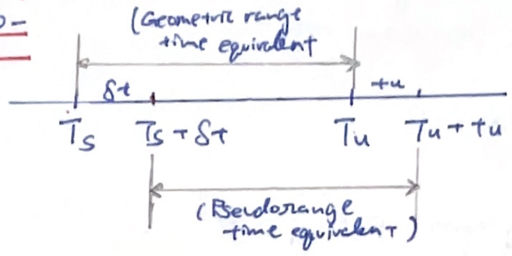
△ factors that effect Satellite orbit/clock

- perturbation forces
  - non sphericity of earth's gravitational potential
  - 3rd body effects
  - solar radiation pressure.
- relativity effects — less gravity → time faster
  - — faster speed → time slower

Ch2 Positioning

TOA & GNSS measurement

pseudo-range



$$\rho = c\left[(T_u+t_u)-(T_s+\delta t)\right]+\sum\delta$$

$$= \underbrace{c(T_u-T_s)}_{\text{Geometric Range}} + \underbrace{c(t_u-\delta t)}_{\text{Clock offset from system time}} + \underbrace{\sum\delta}_{\text{delay \& noise}}$$

$\sum\delta$ = Trop + Ion + K_{rec} + K^{sat} + ε

- T_u+t_u , $T_s+\delta t$ are obtained from
  - Ranging codes &
  - non-sychronized clock

carrier phase

$$\rho = \lambda\cdot\Delta\varphi = \lambda(\underbrace{\varphi_r(T_2)}_{\text{phase of signal reception}} - \underbrace{\varphi^s(T_1)}_{\text{phase of signal transmission}})$$

- much more precise
- yet w/ ambiguity on integer of existing wavelengths

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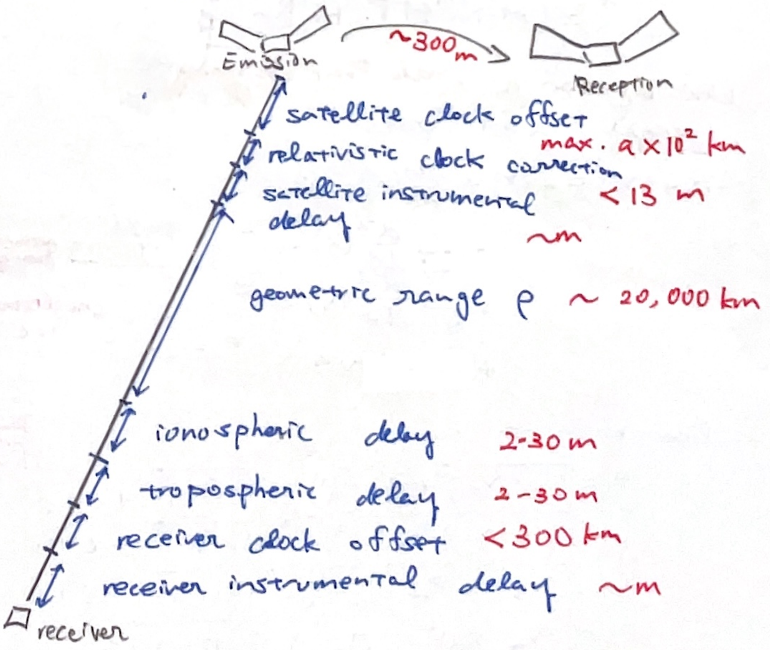
pseudo-range error source



Least Squares

$x \in R^n$ (tracking state)

$y \in R^m$ (observation state

$$\begin{cases} y = Hx + n \\ \hat{x} = \underset{x}{\operatorname{argmax}} \; P(y|x) \end{cases}$$

$n \sim N(0, \sigma)$

$$\therefore \hat{x} = \underset{x}{\operatorname{argmax}} \frac{1}{(2\pi\sigma)^{N/2}} e^{-\frac{1}{2\sigma^2}\underline{\|y-Hx\|^2}}$$

$$= \underset{x}{\operatorname{argmin}} \|y - Hx\|^2$$

$$\therefore \text{ we set } \frac{d}{d\hat{x}} \underline{\|y - H\hat{x}\|^2} = 2H^T H\hat{x} - 2H^T y$$

$$\therefore \hat{x} = (H^T H)^{-1} H^T y$$

△ weighted version

$$\hat{x} = \underset{x}{\operatorname{argmax}} \frac{1}{(2\pi)^{\frac{N}{2}} |R_n|^{\frac{1}{2}}} e^{-\frac{1}{2}(y-Hx)^T R_n^{-1}(y-Hx)}$$

$$= \underset{x}{\operatorname{argmin}} (y-Hx)^T R_n^{-1} (y-Hx)$$

$$\therefore \hat{x} = (H^T R_n^{-1} H)^{-1} H^T R_n^{-1} y$$

linearization (Single Point Positioning)

△ recall Taylor Series

$$f(x+\Delta x) = f(x) + \frac{df}{dx}(x) \cdot \Delta x + \frac{1}{2!}\frac{d^2 f}{dx^2}(x)\Delta x^2 + \frac{1}{3!}\frac{d^3 f}{dx^3}(x)\Delta x^3 + O(\Delta x^4)$$

recall pseudo-range

$$\rho = \underbrace{c(T_u - T_s)}_{\text{geometry range}} + c(t_u - \delta t) + \Sigma\delta \text{ (neglected here)}$$

$$= r + c(\underbrace{t_u}_{\text{receiver offset}} - \underbrace{\delta t}_{\text{satellite offset}})$$

$$= r + ct_u$$

~~known~~ here (broadcasted)

△ linearization to solve

$$\rho_1 = \sqrt{(x_1 - \boxed{x_u})^2 + (y_1 - \boxed{y_u})^2 + (z_1 - \boxed{z_u})^2} + c\boxed{t_u}$$

$$\rho_2 = \sqrt{(x_2 - x_u)^2 + (y_2 - y_u)^2 + (z_2 - z_u)^2} + ct_u$$

$$\rho_3 = \sqrt{(x_3 - x_u)^2 + (y_3 - y_u)^2 + (z_3 - z_u)^2} + ct_u$$

$$\rho_4 = \sqrt{(x_4 - x_u)^2 + (y_4 - y_u)^2 + (z_4 - z_u)^2} + ct_u$$

known known

4 unknowns

$$\triangle \rightarrow \rho_j = \sqrt{(x_j - x_u)^2 + (y_j - y_u)^2 + (z_j - z_u)^2} + ct_u$$

$$= f(x_u, y_u, z_u, t_u)$$

assume a known, approximate position location $(\hat{x}_u, \hat{y}_u, \hat{z}_u)$ & $\hat{t}_u$ (time bias

$$\hat{\rho}_j = \sqrt{(x_j - \hat{x}_u)^2 + (y_j - \hat{y}_u)^2 + (z_j - \hat{z}_u)^2} + c\hat{t}_u$$

$$= f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u) \quad \text{— } \circledast$$

$$\triangle \rightarrow x_u = \hat{x}_u + \Delta x_u \qquad z_u = \hat{z}_u + \Delta z_u$$

$$y_u = \hat{y}_u + \Delta y_u \qquad t_u = \hat{t}_u + \Delta t_u$$

$$\therefore f(x_u, y_u, z_u, t_u) = f(\hat{x}_u + \Delta x_u, \hat{y}_u + \Delta y_u, \hat{z}_u + \Delta z_u, \hat{t}_u + \Delta t_u)$$

$\therefore$ TS of $f(x_u, y_u, z_u, t_u)$:

$$f(x_u, y_u, z_u, t_u) = f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u) + \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{x}_u}\Delta x_u + \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{y}_u}\Delta y_u + \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{z}_u}\Delta z_u + \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{t}_u}\Delta t_u + O(\Delta^2) \quad \text{— } \otimes$$

(from above)

$$\left.\begin{aligned} \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{x}_u} &= -\frac{x_j - \hat{x}_u}{\hat{r}_j} \\ \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{y}_u} &= -\frac{y_j - \hat{y}_u}{\hat{r}_j} \\ \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{z}_u} &= -\frac{z_j - \hat{z}_u}{\hat{r}_j} \\ \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{t}_u} &= c \\ \hat{r}_j &= \sqrt{(x_j - \hat{x}_u)^2 + (y_j - \hat{y}_u)^2 + (z_j - \hat{z}_u)^2} \end{aligned}\right\} \#$$

△ put # & $\circledast$ into $\otimes$

we get

$$\rho_j = \hat{\rho}_j - \overbrace{\frac{x_j - \hat{x}_u}{\hat{r}_j}}^{a_{x_j}}\Delta x_u - \overbrace{\frac{y_j - \hat{y}_u}{\hat{r}_j}}^{a_{y_j}}\Delta y_u - \overbrace{\frac{z_j - \hat{z}_u}{\hat{r}_j}}^{a_{z_j}}\Delta z_u + c\Delta t_u$$

$$\Rightarrow \rho_j - \hat{\rho}_j = -a_{x_j}\Delta x_u - a_{y_j}\Delta y_u - a_{z_j}\Delta z_u + c\Delta t_u$$

$$\Rightarrow \Delta \rho_j = a_{x_j}\Delta x_u + a_{y_j}\Delta y_u + a_{z_j}\Delta z_u - c\Delta t_u$$

$$\Rightarrow \begin{aligned} \Delta\rho_1 &= a_{x_1}\Delta x_u + a_{y_1}\Delta y_u + a_{z_1}\Delta z_u - c\Delta t_u \\ \Delta\rho_2 &= a_{x_2}\Delta x_u + a_{y_2}\Delta y_u + a_{z_2}\Delta z_u - c\Delta t_u \\ \Delta\rho_3 &= a_{x_3}\Delta x_u + a_{y_3}\Delta y_u + a_{z_3}\Delta z_u - c\Delta t_u \\ \Delta\rho_4 &= a_{x_4}\Delta x_u + a_{y_4}\Delta y_u + a_{34}\Delta z_u - c\Delta t_u \end{aligned}$$

$$\Rightarrow \Delta\rho = \begin{bmatrix} \Delta\rho_1 \\ \Delta\rho_2 \\ \Delta\rho_3 \\ \Delta\rho_4 \end{bmatrix} \quad H = \begin{bmatrix} a_{x_1} & a_{y_1} & a_{z_1} & 1 \\ a_{x_2} & a_{y_2} & a_{z_2} & 1 \\ a_{x_3} & a_{y_3} & a_{z_3} & 1 \\ a_{x_4} & a_{y_4} & a_{34} & 1 \end{bmatrix} \quad \Delta x = \begin{bmatrix} \Delta x_u \\ \Delta y_u \\ \Delta z_u \\ -c\Delta t_u \end{bmatrix}$$

$$\Rightarrow \Delta\rho = H\Delta x \longrightarrow \text{could be } > 4 \text{ over-determined}$$

- get Δx (can use least-square)
- get $(x_u, y_u, z_u, t_u) = (\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u) + \Delta x$

Dilution of Precision (DOP)

$\frown \sqrt{D_{11}+D_{22}+D_{33}+D_{44}}$

- PDOP/GDOP $= \sqrt{D_{11}+D_{22}+D_{33}}$
- HDOP $= \sqrt{D_{11}+D_{22}}$
- VDOP $= \sqrt{D_{33}}$
- TDOP $= \sqrt{D_{44}}/c$

$$H\Delta x = \Delta P$$

$$H = \begin{bmatrix} a_{x1} & a_{y1} & a_{z1} & 1 \\ a_{x2} & a_{y2} & a_{z2} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ a_{xn} & a_{yn} & a_{zn} & 1 \end{bmatrix}$$

$$\Delta X = (H^T H)^{-1} H^T \Delta P$$

$$\mathrm{cov}(dx) = (H^T H)^{-1} \sigma^2_{UERE}$$

user equivalent range error

$$(H^T H)^{-1} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} \\ D_{21} & D_{22} & D_{23} & D_{24} \\ D_{31} & D_{32} & D_{33} & D_{34} \\ D_{41} & D_{42} & D_{43} & D_{44} \end{bmatrix}$$

→ better geometry , less DOP

→ VDOP > HDOP , no variation vertically

→ precise → concentrated data
accurate → ground-truth-closed data

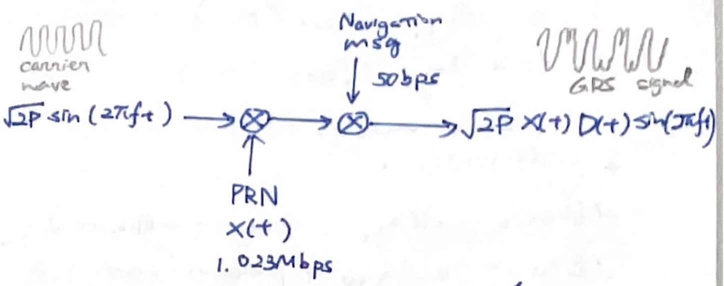
SV clock correction

- satellite clock offset
  - monitored by groud station network
  - rebroadcasted to receiver
- clock correction model
  - thru polynomial
  - polynomial coefficients a_0, a_1, a_2
  - prediction model (hours to days)
  - reference by T_{oc} → operation center

Ch3 GPS/GNSS signal

Signal Structure

- signal = carrier · wave + navigation msg (ephemeris, satellite clock/correction) / PRN code (for Time of Arrival)



- carrier wave @ 1575.42MHz , L1 , is modulated by
  - coarse/acquisition (C/A) code civilians
  - precision/ secure (P/Y) code military

PRN (Pseudo Random Noise) Code

- for measurement of time of arrival (TOA)
- generation pattern : Gold Code
- 1 second : 50 bit
  - → 1000 ms : 50 bit
  - → 20 ms : 1 bit
  - → 20 ms : 20 repetition of C/A
  - → 1 ms : 1 repetition of C/A
  - → 1 repetitiof C/A : 1023 $\frac{1}{0}$
- C/A : 1.023 M cps
  P/Y : 10.23 M cps

Signal @ L-band lower frequency gets easier to be affected by ionosphere

Auto-correlation

① Satellite transmits code-phase with timestamp $T^S(n)$

② Receiver receives code-phase with time stamp $T_R(n)$
n here stands for epoch n

③ on timeline of " Receiver ", a replica code-phase is generated at $T^S(n)$ on timeline "receiver"

☆ ④ conduct auto-correlation by shifting the replica code-phase to match received code-phase on timeline receiver

⑤ the shifted Δt is the inferred propagation time

⑥ pseudorange = $c \cdot \Delta t$

⑦ this pseudorange =

$$c\left[(T_u+t_u)-(T_s+\delta t)\right] + \Sigma\delta$$

Carrier Phase : resolve fractional difference / integer ambiguity (N)
could get accuracy of 1-2cm

Power of GPS

- Bel $B = \log_{10}(P_{out}/P_{in})$
- Decibel $dB = \log_{10}(P_{out}/P_{in})$
- set reference value (P_{in})
  1 milliwatt :

$dBm = 10 \log_{10}(\text{power in milliwatt}) / 1 \text{ milliwatt}$

$dBW = 10 \log_{10}(\text{power in watt}) / 1 \text{ WATT}$

△ Path Loss

- transmit power is 27W
- received power reduced by 10^{16}

-130 dBm ~ -160 dBm < Thermal Noise Power -110 dBm < WIFI BT Phones

△ Antenna Gain : ratio of $\frac{\text{entire sphere area}}{\text{spot area}}$

Ch4 Differential Positioning & Precise Positioning

Errors

$$\delta t_D = \delta t_{atm} + \delta t_{\substack{noise\\inter.}} + \underset{\text{multi-path}}{\delta t_{mp}} + \underset{\text{hardware}}{\delta t_{hw}}$$

- atmosphere
- receiver noise + interference
- multipath
- receiver hardware offset

△ Satellite Clock Error

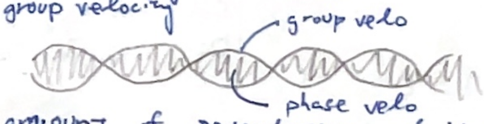
- offset of SV time to GPS time
- curve-fit to model the clock correction (polynomial coefficients in navmsg)
- not location-dependent for users

△ Ephemeris Error

- due to perturbation forces
- curve-fit to model the error
- location-dependent for users
  (increases with separation)
  base station receiver

△ Ionosphere Delay

- 電離層
- affects electromagnetic wave propagation
- dispersive medium 15XX MHz
  - $f > f_p \simeq 10^6$ Hz can travel thru
  - phase velocity will exceed group velocity



  - amount of retardation of Vgroup
    = advance of carrier phase w.r.t. free-space propagation
  - GPS :
    PRN & navmsg delayed than carrier phase
  → ionospheric divergence
- removed by dual receiver
- depend on frequency f ,
  elevation angle φ' (@ iono. pierce pt) ,
  total electron content TEC
- location-dependent for users
  (increases w/ separation ,
  base station receiver

△ Troposphere Delay

- 對流層
- nondispersive, but with refraction
-

dry (hydrostatic)	wet (nonhydrostatic)
- 90% to delay	- lesser to delay
- predict accurately	- difficult to predict
- extends to 40km height	- extends to 10km height
- utilize troposphere model to predict	

basically depends on weather

- location-dependent for users

△ Receiver Noise
- thermal noise jitter
- interference/jamming (malicious signal)

△ Multipath
- 
- depends on environment
  - receiver location
  - satellite elevation angle
  - receiver signal processing
  - antenna gain pattern
  - signal characteristic
- improvements :
  - detect → received signal pattern thru → signal/noise density ratio to see → reflection/refraction
  - signal processing : reduce early/late correlator
  - antenna : increase directivity

△ Hardware Bias
- Satellite Bias
  - signals and carrier frequency not totally synced.
- Receiver/Equipment Bias
  - signals are delayed in hardware

1. From above, usually (single point), dominan pseudo-range error : "Ionospheric Delay"

→ could be removed w/ dual receiver

2. spatially correlated :
- Ephemeris Errors
- Ionospheric Error
- Tropospheric Error
- multipath

Real-Time Kinematic (RTK)

- base receiver + rover receiver (≥2)
- base receiver process GPS signals
- rover receiver process GPS signals
- base station broadcast its measurement rover receiver
- rover receiver could then resolves some error like ionospheric errors
- harnesses both pseudo-range (code) & carrier phase
- m → cm

★ thru base correction & RTK algo m → cm

★ single-band baseline <10km
multi-band baseline <60 km

★ base unit could be provided thru "local service provider"

△ typical algo.

1. KF
2. unknowns : X_rover, V_rover, ambg.
3. measurements : double differenced carrier phase
   double differenced pseudo-range
4. Integer ambg. solution : a. EKF update
   b. solve ambg. improve [accuracy / convergence time]

Precise Point Positioning (PPP)

- single receiver
- relies on precise satellite clock / orbit products correction
  ↑ from global stations.
- cm-level.
- PPP performace > RTK

Ch5 Augmentation Systems

differential positioning (relative positioning)

- 2 receivers are close
- track difference between the 2
  - can be tracked more accurately
  - iono + tropo + clock + ephemeris errors are cancelled.
- ΔX = (P₁, t₁) − (P₂, t₂)
  ↳ know ΔX, know reference get rover.
- base station, with high-performance GPS receiver, could calculate error/correction & send correction to rover.
- DGPS = reference station + data link + user receiver
- Local Area Differential GPS
  Wide Area Differential GPS

Ground Based Augmentation Systems (GBAS)

- pseudo-range corrections
- use / don't use
- Aviation
- DGPS

Satellite Based Augmentation Systems (SBAS)

- Aviation
- DGPS
- iono., clock, ephimeris.
- GEO
- monitored / not monitored
- use / don't use

→
- send corrections
- send integrity msgs.