No

Date

ME564 L3

1. Taylor series & $\dot{x} = \lambda x$

2. What is Taylor series

3. 2^{nd} order systems

$$\ddot{x} + x = 0$$

- harmonic oscillator, spring-mass
- try taylor series

$\dot{x} = ax \Rightarrow x(t) = e^{at}x(0)$

use Taylor Series : → To derive

higher order terms

$$x(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3 + \dots + h.o.t$$

$O(t^4)$ w/ $x(0) = x_0$

$$\dot{x}(t) = 0 + C_1 + 2C_2 t + 3C_3 t^2 + 4C_4 t^3 + \cdots \quad \therefore C_0 = x_0$$

$$a x = aC_0 + aC_1 t + aC_2 t^2 + aC_3 t^3 + \cdots$$

$C_0 = x_0$

$t^0 \Rightarrow \quad C_1 = aC_0 \quad C_1 = ax_0$

$t^1 \Rightarrow \quad 2C_2 = aC_1 \quad C_2 = \frac{a^2}{2}x_0$

$t^2 \Rightarrow \quad 3C_3 = aC_2 \quad C_3 = \frac{a^3}{3!}x_0$

$t^3 \Rightarrow \quad 4C_4 = aC_3 \quad C_4 = \frac{a^4}{4!}x_0$

$\vdots$

$$C_N = \frac{a^n}{N!}x_0$$

$$x(t) = x_0 + x_0 at + \frac{a^2}{2}t^2 x_0 + \frac{a^3}{3!}t^3 x_0 + \dots$$

$$= \underbrace{\left(1 + at + \frac{(at)^2}{2!} + \frac{(at)^3}{3!} + \dots\right)}_{e^{at}} x_0$$

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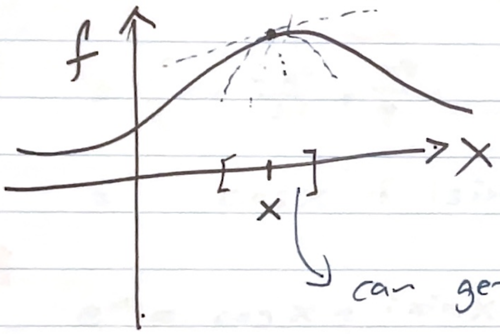
$\dot{x} = Ax \qquad x(t) = e^{At} x_0$

derivatives exist

Taylor Series — smooth function could all be expanded as T.S.

: A function $f(x+\Delta x)$ can be Taylor expanded at

$$f(x+\Delta x) = f(x) + \frac{df}{dx}(x)\cdot\Delta x + \frac{1}{2!}\frac{d^2 f}{dx^2}(x)\cdot\Delta x^2 + \frac{1}{3!}\frac{d^3 f}{dx^3}(x)\,\Delta x^3 + \dots + \frac{1}{n!}\frac{d^n f}{dx^n}(x)\,\Delta x^n + \dots$$



can get good approximation on the neighborhood.

Equivalent

other representation

$f(x)$ expanded about a base pt. "a"

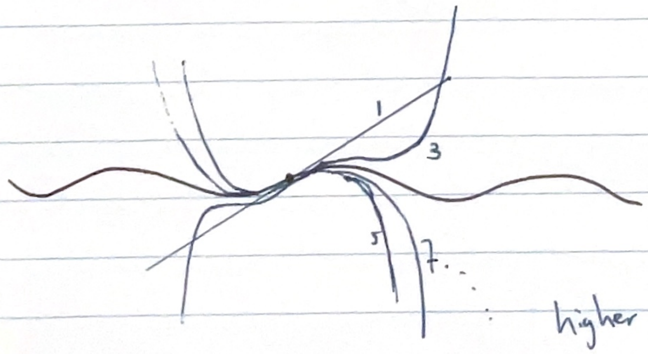
$$f(x) = f(a) + \frac{df}{dx}(a)\underbrace{(x-a)}_{\Delta x} + \frac{d^2 f}{dx^2}(a)(x-a)^2 + \dots$$

e.g. $f(x) = \sin(x)$ about $a = 0$ (maclaren Series)

$$= \cancel{\sin(0)} + \cos(0)\cdot x + \cancel{-\sin(0)\cdot\frac{x^2}{2!}} - \cos(0)\frac{x^3}{3!} + \cancel{\sin(0)\frac{x^4}{4!}} + \cos(0)\frac{x^5}{5!} + \dots$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$f(x) = \cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$



higher order, higher precision in approximation

ME564 L3 (cont'd)

generalizing complex number w/ Taylor series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \dots$$

$$e^{ix} = 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \dots$$

$i = \sqrt{-1}$

$$= 1 + ix + \frac{-x^2}{2!} + \frac{-ix^3}{3!} + \frac{x^4}{4!} + \frac{ix^5}{5!} + \dots$$

$$= \left(1 + \frac{-x^2}{2!} + \frac{x^4}{4} + \dots\right) + i\left(ix - \frac{ix^3}{3!} + \frac{ix^5}{5!} + \dots\right)$$

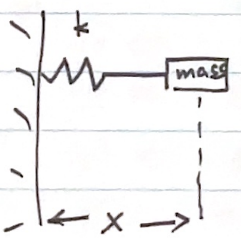
$$= \cos x + i \sin x$$

☆

$\therefore$ $e^{ix} = \cos x + i \sin x$ Euler's Formula

Second Order ODEs :

$\ddot{x} + x = 0$ $\ddot{x} + \dot{x} + x = 0$



$F = ma$

$F = -kx$

$\triangle$ $-kx = m\ddot{x}$

$\ddot{x} = \frac{-k}{m} x$

method 1 :
guess a solution

method 2 :
also guess
$x(t) = e^{\lambda t}$ → plug this into equation

↓

generalize