ME564 L26

$$\left.\begin{aligned} &\dot{x} = Ax \\ &\dot{x} = f(x) \\ &x(t) \end{aligned}\right\} \text{ODEs}$$

$\dot{x} = Ax$

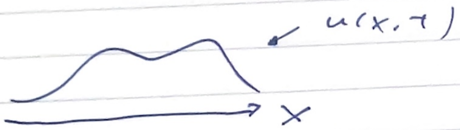
$z = Tx$ change of coords.

$\dot{z} = T^{-1}ATz$

$= Dz$

$$\left.\begin{aligned} &\frac{\partial u}{\partial t} = c\frac{\partial u}{\partial x} \\ &u(x,t) \end{aligned}\right\} \text{PDEs}$$

Fourier
Laplace. change of coords



Potential Flow

consider a fluid, which is incompressible, & irrotational $\rightarrow \vec{V}(x,t)$ let $\vec{V} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

① steady : $\frac{\partial \vec{V}}{\partial t} = 0 \longrightarrow \therefore \vec{V}(x)$

② incompressible: $\nabla \cdot \vec{V} = 0$ divergence $= 0$ $\quad \frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} = 0$

③ irrotational: $\nabla \times \vec{V} = 0$ curl $= 0$ $\quad \frac{\partial V_2}{\partial x} - \frac{\partial V_1}{\partial y} = 0$

simple PDE
we can solve!

$$\boxed{\begin{aligned} &\text{if } \vec{V} = \nabla \varphi \quad \text{is a scalar function / scalar field} \\ &\& \ \varphi \text{ satisfies Laplace's Equation } \nabla^2\varphi = 0 \\ &\text{then } \vec{V} \text{ satisfies ② \& ③} \end{aligned}}$$

$$\boxed{\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0}$$

PDE

∇^2 is called "The Laplacian"

$$\nabla^2\varphi = \nabla \cdot (\nabla \varphi)$$

$$= \nabla \cdot \begin{bmatrix} \partial\varphi/\partial x \\ \partial\varphi/\partial y \end{bmatrix} = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2}$$

No.

Date

(cont'd)

$\nabla \times (\nabla \varphi) = 0$

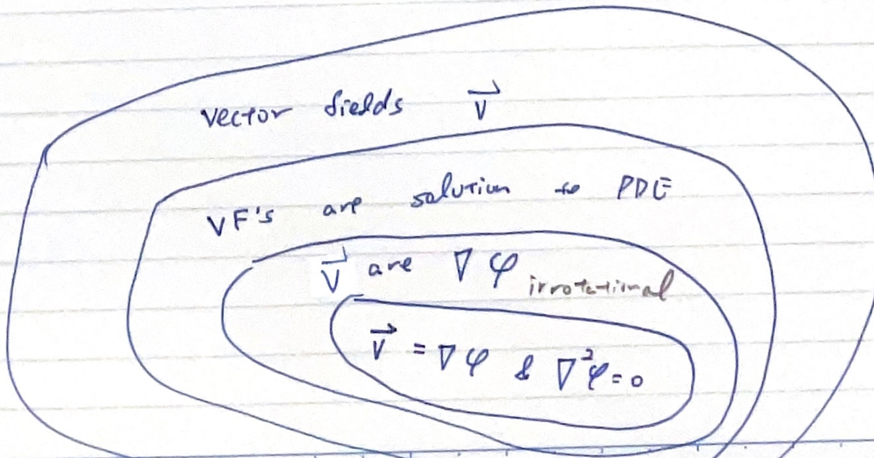
for all φ

$$\vec{V} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \nabla \varphi = \begin{bmatrix} \frac{\partial \varphi}{\partial x} \\ \frac{\partial \varphi}{\partial y} \end{bmatrix}$$

incomprissible
$$\nabla \cdot \vec{V} = \frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y}$$
$$= \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2}$$
$$= 0$$
laplacian

irrotational
$$\nabla \times \vec{V} = \frac{\partial V_2}{\partial x} - \frac{\partial V_1}{\partial y}$$
$$= \frac{\partial^2 \varphi}{\cancel{\partial y}\, \partial x} - \frac{\partial^2 \varphi}{\cancel{\partial x}\, \partial y}$$
$$= 0$$

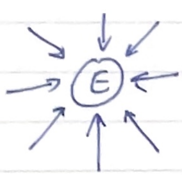
Airplanes before computers



Laplace's Equation

$$\boxed{\nabla^2 \varphi = 0}$$

① Gravitation (away from mass sources)



$\vec{F} = -\nabla p$ where $p = \frac{-mMG}{r}$

verify that $\nabla^2 p = 0$ (away from mass sources)

② Electrostatics (away from point charges)

③ heat conduction (steady - state) T (temperature)

$\frac{\partial T}{\partial t} = c^2 \nabla^2 T \longrightarrow \nabla^2 T = 0$

$t \to \infty$

$T = k$ this satisfy $\nabla^2 T = 0$

④ potential flow