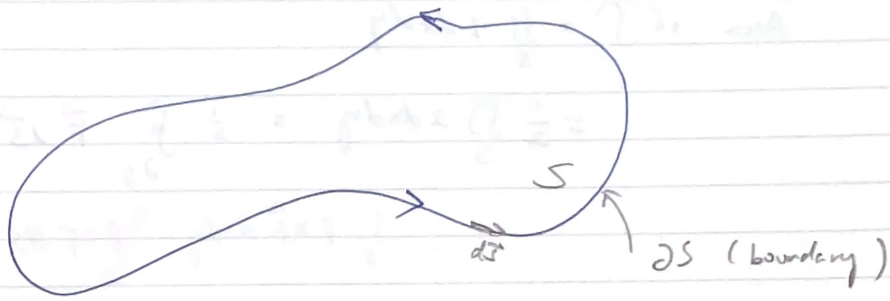
ME564
L25

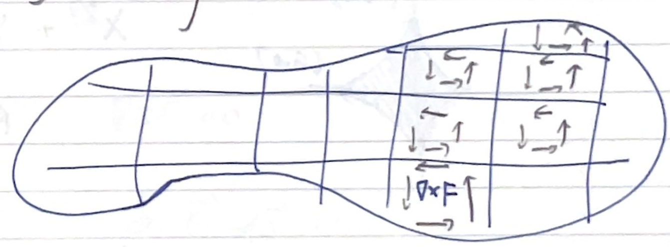
Stoke's Theorem



$$\oint_{\partial S} \vec{F} \cdot d\vec{s} = \iint_S \nabla \times \vec{F} \cdot ds$$

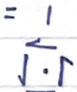
→ curl from interior boxes cancelled out
→ left with tangent surfaces vectors

Stokes theorem



$\nabla \cdot \vec{F} = 1$

$\nabla \times \vec{F} = 1$



in 2D

Green's Theorem

$\vec{F} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$ scalar function / scalar function.

vector function

$$\int_{\partial S} \vec{F} \cdot ds$$

$$= \int_{\partial S} F_1 dx + F_2 dy. = \iint_S \nabla \times \vec{F} \cdot ds = \iint \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dx\, dy$$

No.

Date

1st Use : compute area

let $\vec{F} = \begin{bmatrix} -y \\ x \end{bmatrix}$ $\therefore$ $\nabla \times \vec{F} = 2$

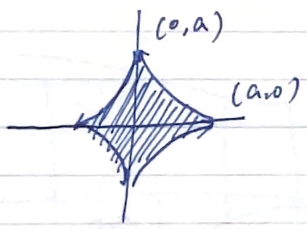
Area of $S = \iint_S 1 \, dx\,dy$

$= \frac{1}{2} \iint_S 2 \, dx\,dy = \frac{1}{2} \oint_{\partial S} \vec{F} \, d\vec{s} = \frac{1}{2} \oint_{\partial S} -y\,dx + x\,dy$

$\iint_S \nabla \times \vec{F} \, dx\,dy = \oint_{\partial S} \vec{F} \, ds$

$\therefore$ Area of $S = \frac{1}{2} \oint_{\partial S} x\,dy - y\,dx$

e.g. Hypocycloid. areas



$x^{2/3} + y^{2/3} = a^{2/3}$

parameterize :

$x = a\cos^3(\theta)$

$y = a\sin^3(\theta)$

$\theta = 0 \to 2\pi$

$dx = -3a\cos^2(\theta)\sin(\theta)\,d\theta$

$dy = 3a\sin^2(\theta)\cos(\theta)\,d\theta$)

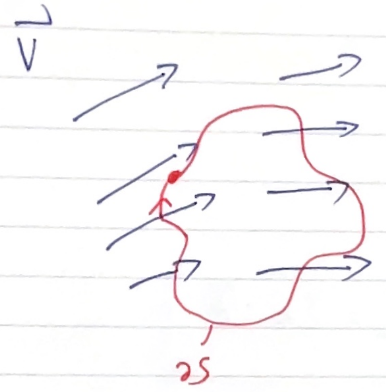
$A = \frac{1}{2} \int_{\partial A} x\,dy - y\,dx$

$= \frac{1}{2} \int_0^{2\pi} \left[3a^2\sin^2\cos^4 + 3a^2\sin^4\cos^2 \right] d\theta$

$= \frac{3}{8}\pi a^2$

2nd use : Physics w/ rotation. e.g. ◦ Kelvin's circulation theorem
◦ lift on helicopter

3rd use , Physics w/o rotation



no curl

$$\oint_{\partial S} \vec{F}\cdot ds = \iint_{\partial} \nabla\times\vec{F}\, dx\, dy = 0$$

if $\nabla\times\vec{F} = 0$

then $\oint_{\partial S} \vec{F}\cdot d\vec{s} = 0$

for all closed curves "∂S"

then $\vec{F}$ is called a (conservative vector field!!!)

e.g. gravitational force field

$$\nabla\times(\underbrace{\nabla\varphi}_{\vec{F}}) = 0$$

$$\vec{F} = \begin{bmatrix} \partial\varphi/\partial x \\ \partial\varphi/\partial y \\ \partial\varphi/\partial z \end{bmatrix} = \frac{\partial\varphi}{\partial x}\vec{i} + \frac{\partial\varphi}{\partial y}\vec{j} + \frac{\partial\varphi}{\partial z}\vec{k}$$

dx dy dz

if $\vec{F} = \nabla\varphi$, then $\vec{F}$ is conservative

(if vector field is a gradient of some scalar function)

$$\oint_C \vec{F}\cdot d\vec{s} = 0$$