No.
Date

MES64 L22

vector calculus

Div, Grad. Curl

$\nabla \cdot \vec{f}$ Div

∇f Grad

$\nabla \times \vec{f}$ Curl

∇ Del, Nabla & vectors of Partial Derivatives

$$\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z} = \begin{bmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{bmatrix}$$

∇ is a linear operator

$$\nabla(f+g) = \nabla f + \nabla g \qquad f(x,y,z)$$

$$\nabla(\alpha f) = \alpha \nabla f \qquad g(x,y,z)$$

α is a number

$$[A][V_1 + V_2]$$

matrix multip. is linear operator.

Gradient

$\nabla f(x,y,z)$ is called gradient.

how fast my function is changing.

$$\nabla f = \begin{bmatrix} \partial f/\partial x \\ \partial f/\partial y \\ \partial f/\partial z \end{bmatrix}$$

input
scalar function

output
vector field

Ex. 1.

Mass in Gravity Field

mass of object

mass of earth

universal gravity constant

Newtons Law: $\vec{F} = -\frac{mMG}{r^3}\vec{r}$

$= -\nabla V$ (potential field)

Gravitational Potential.

$$V = -\frac{mMG}{r} \quad \text{(scalar)}$$

$$= \frac{-mMG}{\sqrt{x^2+y^2+z^2}}$$

linear about earth radius.

$\frac{1}{r_0+h}$ (linearized)

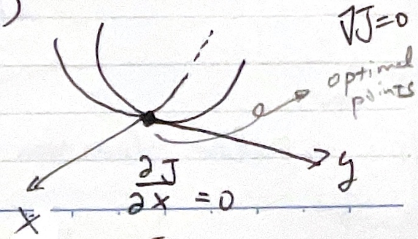
$$\frac{\partial V}{\partial x} = \frac{mMGx}{(x^2+y^2+z^2)^{3/2}}$$

$$\frac{\partial V}{\partial y} = \frac{mMGy}{(x^2+y^2+z^2)^{3/2}}$$

$$\frac{\partial V}{\partial z} = \frac{mMGz}{(x^2+y^2+z^2)^{3/2}}$$

$$\vec{F} = -\nabla V = \begin{bmatrix} -m\frac{MGx}{r^3} \\ -m\frac{MGy}{r^3} \\ -m\frac{MGz}{r^3} \end{bmatrix}$$

$J(x,y)$



$\nabla J = 0$

$\frac{\partial J}{\partial y} = 0$

Divergence.

input vector field
output scalar

Since ∇ is a vector, I can take inner product w/ $\vec{f}$

$$\underbrace{\vec{f}(x, y, z)}_{\text{vector field}} = \begin{bmatrix} f_1(x, y, z) \\ f_2(x, y, z) \\ f_3(x, y, z) \end{bmatrix} \quad (\text{is still a fuction})$$

$\nabla \cdot \vec{f}$ is the divergence of $\vec{f}$

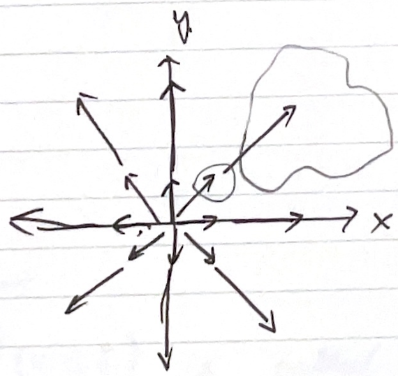
$$\nabla \cdot \vec{f} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

How much will this vector field diverge.

Ex 1.

$$\vec{f} = x\vec{i} + y\vec{j}$$

$$\nabla \cdot \vec{f} = 1 + 1 = 2$$

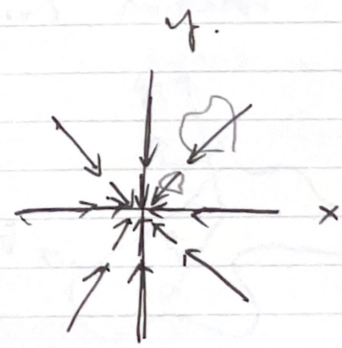


flow is diverging

Ex 2.

$$\vec{f} = -x\vec{i} + -y\vec{j}$$

$$\nabla \cdot \vec{f} = -2$$

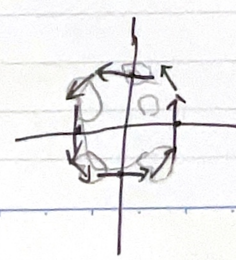


flow is converging.

Ex 3.

$$\vec{f} = -y\vec{i} + x\vec{j}$$

$$\nabla \cdot \vec{f} = 0 + 0 = 0$$



flow is rotation.

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Curl of $\vec{f}$ is $\nabla \times \vec{f}$

$$\nabla_\times = \begin{bmatrix} 0 & -\partial/\partial z & \partial/\partial y \\ \partial/\partial z & 0 & -\partial/\partial x \\ -\partial/\partial y & \partial/\partial x & 0 \end{bmatrix}$$

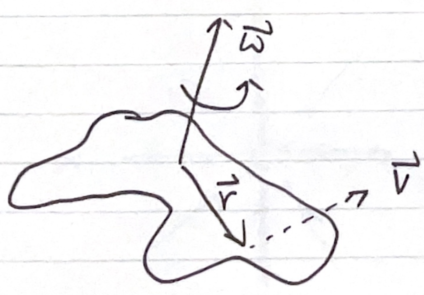
$$\nabla \times \vec{f} = \det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f_1 & f_2 & f_3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -\partial/\partial z & \partial/\partial y \\ \partial/\partial z & 0 & -\partial/\partial x \\ -\partial/\partial y & \partial/\partial x & 0 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

$$\vec{f} = xy\,\vec{i} - \sin(z)\,j + \vec{k}$$

$$\nabla \times \vec{f} = \vec{i}(\cos(z)) - \vec{j}(0) + \vec{k}(-x)$$

measures rotation (vortexity)



$\vec{v} = \vec{\omega} \times \vec{r}$

curl $\vec{v} = 2w$