

ME564 L20

Local $\mathcal{O}(\Delta t^5)$
 global $\mathcal{O}(\Delta t^4)$ \rightarrow proportional to Δt

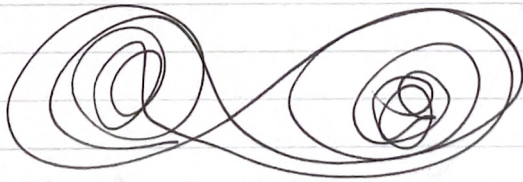
$$\overbrace{\frac{1}{\Delta t}} \times \mathcal{O}(\Delta t^5) \rightarrow \mathcal{O}(\Delta t^4)$$

Not true to Chaotic System!

Integrating Chaotic system, just a little local
 integration error
 could lead to total Chaos (exponential
 growth)

$\xi e^{\lambda t} \rightarrow$ Lyapunov exponent

but still saturated
 at some points
 like Lorenz tucker



$\xi e^{\lambda t}$

$10^{-16} e^t$



machine
 precision

assume $\lambda = 1$

we can get

when $t = ?$

will we ~~can~~ stop
 to tolerate our
 error

How to evaluate precision?

→ out is smaller

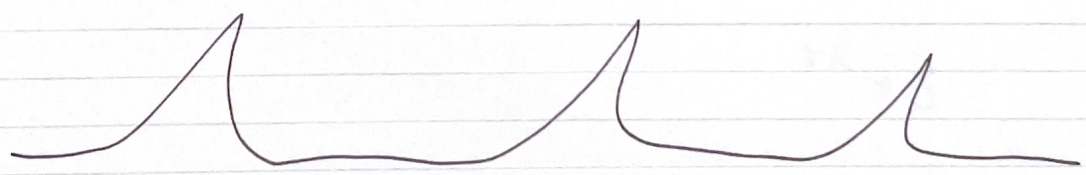
Symplectic
variational Integrators

Hamiltonian
Lagrangian

~~$H = T + V \Rightarrow$~~

$$H(q, p) = T + V \Rightarrow \begin{aligned} \dot{q} &= \frac{\partial H}{\partial p} \\ \dot{p} &= -\frac{\partial H}{\partial q} \end{aligned}$$

maintain some conservation properties



$$\frac{d}{dt} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \omega_1 \\ \omega_2 \end{bmatrix} = f \left(\begin{bmatrix} \theta_1 \\ \theta_2 \\ \omega_1 \\ \omega_2 \end{bmatrix} \right)$$

double pendulum & three-body problem