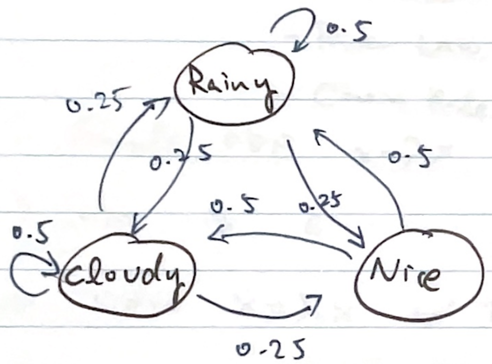
No

Date

ME564 L1

Weather in Seattle



$$X_{today} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} R \\ N \\ C \end{matrix}$$

$$X_{tmr} = A \, X_{today}$$

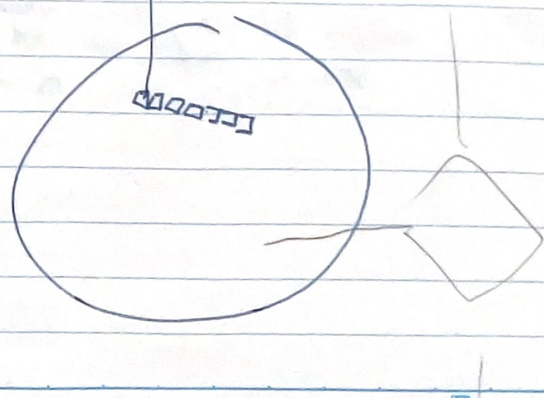
Markov model.

$$A = \begin{array}{cccc} R_{tdy} & N_{tod} & C_{tod} & \\ \left[\begin{matrix} 0.5 \\ 0.25 \\ 0.25 \end{matrix}\right. & \begin{matrix} 0.5 \\ 0 \\ 0.5 \end{matrix} & \left.\begin{matrix} 0.25 \\ 0.25 \\ 0.5 \end{matrix}\right] & \begin{matrix} R_{tmr} \\ N_{tmr} \\ C_{tmr} \end{matrix} \end{array}$$

one pixel

one $X_N = A^{N-1} X_1$

$$X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad X_2 = AX_1 = \begin{bmatrix} 0.5 \\ 0.25 \\ 0.25 \end{bmatrix}, \quad X_3 = A^2 X_1, \quad \cdots \quad X_N = A^{N-1} X_1$$

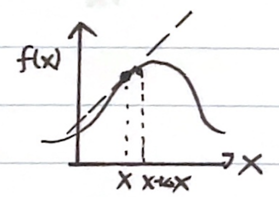


Date

ME564 L2

1. Review Calculus
   - Derivative
   - Power Law
   - Chain Rule
2. Simple ODE $\dot{x} = \lambda x$
3. What is e
4. solve $\dot{x} = \lambda x$ w/ Taylor Series

The Deriviate : rate of change of a function $f(x)$ w.r.t. x



$$\frac{df}{dx} \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\Rightarrow \frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Power Law

$$f(x) = x^n \qquad \frac{df}{dx} = n x^{n-1}$$

$$\frac{df}{dx} \approx \frac{1}{\Delta x}\left[(x+\Delta x)^n - x^n\right]$$

Pascal Triangle
Binomial Expansion

$$= \frac{1}{\Delta x}\left[\cancel{x^n} + n x^{n-1}\Delta x + \frac{n\cdot(n-1)}{2} x^{n-2}\Delta x^2 + \cdots + \cancel{-x^n}\right]$$

$$= \frac{1}{\Delta x}\left[n x^{n-1}\Delta x + \frac{n(n-1)}{2} x^{n-2}\Delta x^2 + \mathcal{O}(\Delta x^3)\right]$$

on the order of Δx cube & higher

$$= n x^{n-1} + \mathcal{O}(\Delta x)$$

on the order of Δx & higher multiplied by at least 1 Δx

$= 0$ when $\Delta x \to 0$

Chain Rule

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x) \qquad f' := \frac{df}{dx}$$

e.g. $\sin(x^3)$

Date

ME564 L2 (cont'd)

X is the size of population of bunnies...

$$\frac{dx}{dt} = \dot{x} = \lambda x$$

↳ change in population, which is proportional to the size of population

ask:

What is the size of pop x as a function of time?

method 1: $\frac{dx}{dt} = \lambda x \Rightarrow \int \frac{dx}{dt} = \int \lambda \, dt$

$$\ln(x(t)) = \lambda t + C$$

$$\therefore x(t) = e^{\lambda t + C}$$

$$= K e^{\lambda t} \Rightarrow \text{get } K \text{ via I.C}$$

initial condition

$$x(t=0) = e^0 K = K$$

$$☆ \quad x(t) = x(0)\, e^{\lambda t}$$

What is e?

Loan amount : x

Loan L — interest rate 'r'

$x(0) = L$

- compound yearly $\longrightarrow x(1) = (1+r)L = (1+r)x(0)$
- compound monthly $\longrightarrow x(1) = (1+\frac{r}{12})^{12} x(0)$

$\downarrow \lim \Delta t \to 0$

$$x(1) = \lim_{n\to\infty} (1+\frac{r}{n})^n x(0)$$

$$= e^r x(0)$$

compound continuously $\exp(0.5) = 1.0513 \longrightarrow$ extra 0.0013!

ME564 L2 (cont'd)

more on e

Interest rate $= r = 0.05$

$n=1$	yearly	$x(1) = (1+0.05)\,x(0)$	$= 1.05\,x(0)$
$n=12$	monthly	$x(1) = (1+\frac{0.05}{12})^{12}\,x(0)$	$= 1.05116\,x(0)$
$n=100$	"	$x(1) = (1+\frac{0.05}{100})^{100}\,x(0)$	$= 1.051257\,x(0)$
$n=10^6$	"	$x(1) = (1+\frac{0.05}{10^6})^{10^6}\,x(0)$	$= 1.051271\,x(0)$
$n\to\infty$ continuously	"	$x(1) = (1+\frac{0.05}{n})^{n}\,x(0)$	
		$= e^{r}x(0)$	
		$= e^{0.05}\,x(0)$	$= 1.051271\,x(0)$

for sth system w $\dot{x} = \lambda x$

$$\frac{dx}{dt} = r\,x(t) \qquad \text{continuous}$$

$$\Delta x = r\,x(t)\,\Delta t \qquad \text{discrete}$$

---

e.g. Radioactive Decay

$$\dot{x} = -\lambda x(t)$$

rate $\xrightarrow{\text{is}}$ proportional λ to number of $x(t)$.

$$x(t) = e^{-\lambda t}\,x(0)$$

Plutonium half-life = 80 million years $= 8\times10^7$

$$\therefore \quad x(t=80\times10^7) = \frac{\cancel{x(0)}}{2} = e^{-\lambda\, 8\times10^7}\cancel{x(0)}$$

$$\therefore \quad \lambda = \frac{-\ln(1/2)}{8\times10^7} \longrightarrow \text{decay rate no. } \lambda$$

my plutonium

ME564 L2 (cont'd)

No

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Polonium: HL $\approx$ 138 days $\approx$ .35 years

$$\lambda = \frac{-\ln(1/2)}{.35}$$

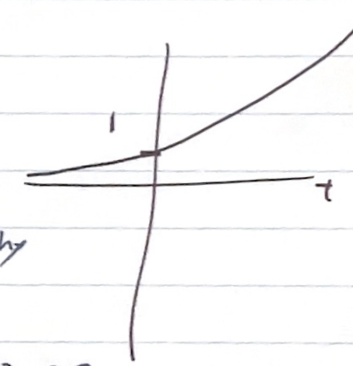
Thermal Runaway lithium-ion for instance

$$\dot{T} = \lambda T \underbrace{-kT^{n}}_{\text{radiating nonlinearity}}$$

$\dot{T} = \lambda T$

$T(t) = e^{\lambda t} T(0)$

my system T should go to infinity, but why doesn't?



fan, water-cooling

computer temperature

radiating nonlinearity

so T does not go to ∞.

$\dot{x} = \lambda x$ $\quad$ $\frac{d}{dx} e^{\lambda x} = \lambda e^{\lambda x}$

$x(t) = C_0 + C_1 t + C_2 t^2 + C_3 C^3 + \dots$

$\Downarrow$

derive $e^{\lambda t}$