ME564 L15

From last time :

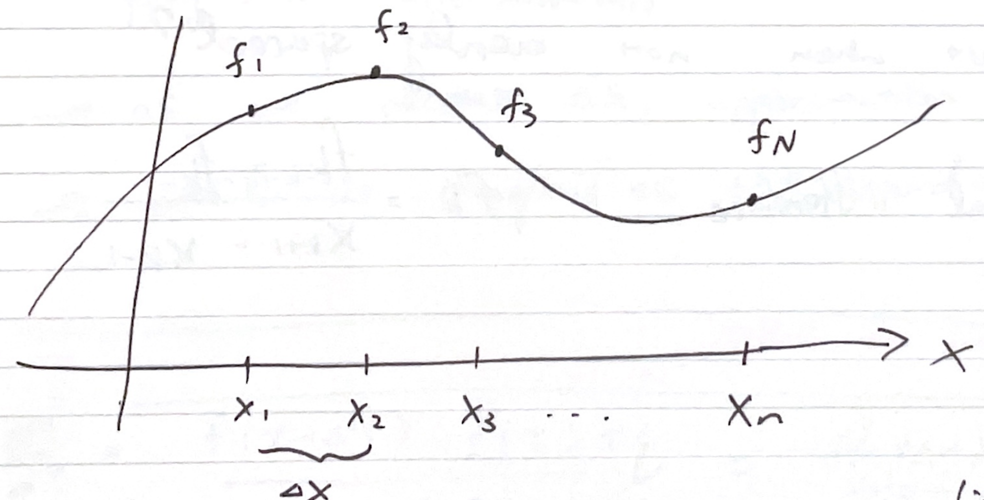
Finite Difference approximation to $f'(x)$

- Forward diff $f'(x) \approx \dfrac{f(x+\Delta x)-f(x)}{\Delta x} + \mathcal{O}(\Delta x)$

- Back diff $f'(x) \approx \dfrac{f(x)-f(x-\Delta x)}{\Delta x} + \mathcal{O}(\Delta x)$

- central diff $f'(x) = \dfrac{f(x+\Delta x)-f(x-\Delta x)}{2\Delta x} + \mathcal{O}(\Delta x^2)$

For 2^{nd} order Diff. : $f(x),\ f(x+\Delta x),\ f(x+2\Delta x)$



discrete
evenly spaced data

$f_k = f(x_k)$
$= f(k\Delta x)$

two vectors of data

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_N \end{bmatrix}$$

calculate $\dfrac{df}{dx}$?

(cont'd) $\dfrac{df}{dx}$?

$$f_k' = \frac{f_{k+1} - f_{k-1}}{2\Delta x}$$

- central difference on the interior pts.
- forward on first pt.
- back on last pt.

How about when not evenly spaced?

central difference : $f'_k = \dfrac{f_{k+1} - f_{k-1}}{x_{k+1} - x_{k-1}}$

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differentiation enlarges error!

numerical smoothing...

Error $\sim \mathcal{O}(\Delta x)$

Compute time $\sim \mathcal{O}\left(\frac{1}{\Delta x}\right)$

$\underline{\underline{10^{-16}}}$ } computer limit precision, ε machine precision

Round-off error

double $\Rightarrow$ 8 bytes

>> $5 + 10^{-17} = 5$ in computer

$(= 5.000\ldots0)$

plug into finite difference

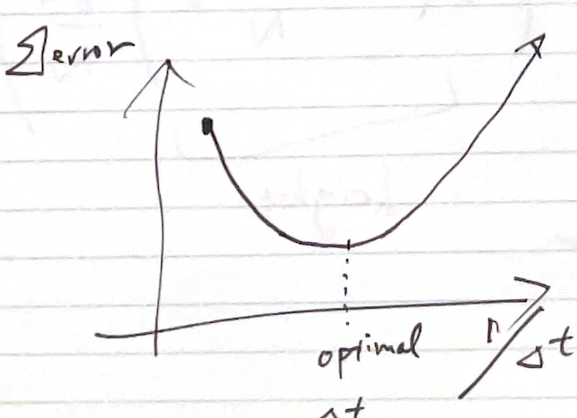
→ as we decrease Δx, approximation gets better

→ until it magnifies the round-up error - worse

→ this one matters

$$\frac{df}{dt} = \frac{f(t+\Delta t) - f(t) + \varepsilon}{\Delta t + \varepsilon} = \frac{f(t+\Delta t) - f(t)}{\Delta t} + \frac{\varepsilon}{\Delta t}$$

$$= \underbrace{\underbrace{f' + \mathcal{O}(\Delta t)}_{\text{taylor series}} + \underbrace{\frac{\varepsilon}{\Delta t}}_{\text{machine precision}}}_{\Sigma \text{ Error}}$$



FD ↓ $\Delta t \approx 10^{-5}$

$(\Delta t \to \text{smaller}) \to \dfrac{\partial \text{Error}(\Delta t)}{\partial \Delta t} = 0$

No.

Date

Integrator !

$$\frac{dx}{dt} = f(x)$$

$$\frac{X_{k+1} - X_k}{\Delta t} = f(x_k)$$

$$\Downarrow$$

$$X_{k+1} = X_k + \Delta t f(X_k)$$

reeman ?

Lebesgue Integral

$$\int_a^b f(x)\,dx = \boxed{\cancel{\lim_{N\to\infty}}} \sum_{k=1}^{N} \left(\underbrace{f\left(a + \frac{b-a}{N}k\right)}_{\text{height}} \frac{b-a}{N} \right)$$

Right sided Rectangle formula

No this in computer

$$= \lim$$

$$\cancel{\lim_{\Delta x \to 0}} \sum_{k=1}^{N} f(x_k)\,\Delta x$$

no this
in numerical
calculus

or

$$\cancel{\lim_{\Delta x \to 0}} \sum_{k=1}^{N} f(x_k)\,(x_k - x_{k-1})$$