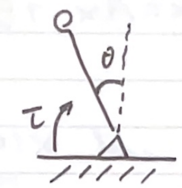
ME564 L13

General Systems $\dot{x} = Ax + Bu$



$$\ddot{\theta} = -\sin(\theta) + \tau$$

linear feedback control system

$$\dot{\theta} = \omega$$
$$\dot{\omega} = -\sin\theta + \tau$$

linearize @ $\theta = \pi$

$$\frac{d}{dt}\begin{bmatrix}\theta \\ \omega\end{bmatrix} = \begin{bmatrix}0 & 1 \\ 1 & 0\end{bmatrix}\begin{bmatrix}\theta \\ \omega\end{bmatrix} + \begin{bmatrix}0 \\ 1\end{bmatrix}\tau$$

$$\dot{x} \quad = \quad A \quad x \quad + \quad B \quad u$$

feedback : $\underbrace{\tau = -2\theta - 2\omega}_{\text{linearized @ } \theta = \pi}$

$$\frac{d}{dt}\begin{bmatrix}\theta \\ \omega\end{bmatrix} = \begin{bmatrix}0 & 1 \\ 1 & 0\end{bmatrix}\begin{bmatrix}\theta \\ \omega\end{bmatrix} + \begin{bmatrix}0 \\ 1\end{bmatrix}\tau$$

$$= \begin{bmatrix}0 & 1 \\ 1 & 0\end{bmatrix}\begin{bmatrix}\theta \\ \omega\end{bmatrix} + \begin{bmatrix}0 & 0 \\ -2 & -2\end{bmatrix}\begin{bmatrix}\theta \\ \omega\end{bmatrix}$$

$$= \begin{bmatrix}0 & 1 \\ -1 & -2\end{bmatrix}\begin{bmatrix}\theta \\ \omega\end{bmatrix} \quad \lambda = -1, -1 \rightarrow \text{stable}$$

No.

Date

General Systems $\dot{x} = Ax + Bu$ solution $x(t) = ?$

Case I:

$u(t) = 0$, and $x(0) = x_0$

$x(t) = e^{At} x_0$ (homogeneous, initial condition response)

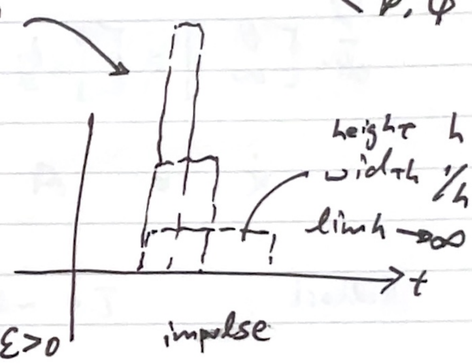
Case 2:

$x(0) = 0$, $u(t) = \delta(t)$, $B = x_0$

↳ Dirac delta function

Dirac

$\langle \phi, \psi \rangle$



impulse function @ $t=5$

$x(0)=0$, $u(t) = \delta(t-5)$

$B = x_0$

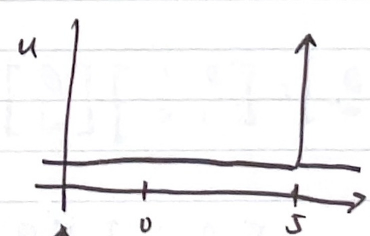
$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

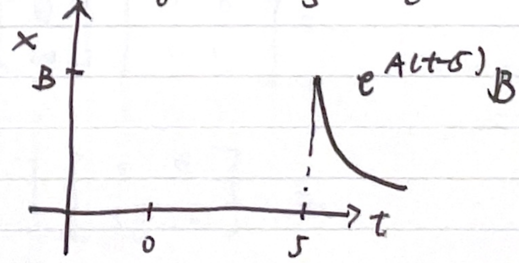
$\int_{-\varepsilon}^{\varepsilon} \delta(t)\,dt = 1$ for any $\varepsilon > 0$

$$x(t) = x(0) + \int_0^t [Ax(\tau) + Bu(\tau)]\,d\tau$$

$x(5^+) = B$

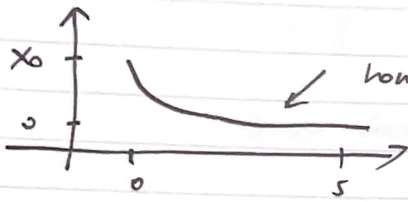
$\Rightarrow x(t) = e^{A(t-5)} B$





Case 3 :

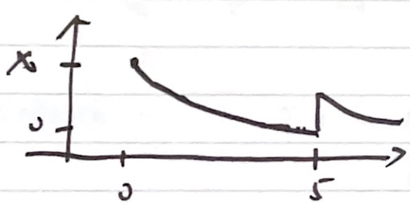
$u(t) = \delta(t-5)$, B, $x(0) = x_0$



homogeneous soln

\+

particular soln due to impulse



full solⁿ!!

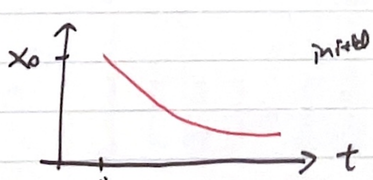
simple addition

General systems $\dot{x} = Ax + Bu$ solⁿ $x(t) = ?$

Convolution

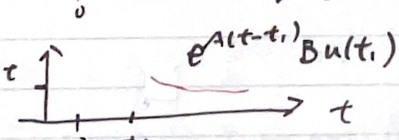
train of weak impulses w/ strength @ each t

$Bu(\tau)\,\delta(\tau)\,d\tau$

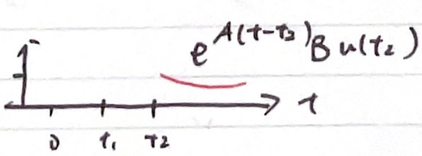


initial condition response

\+ $\uparrow$ $Bu(t_1)d\tau$



\+ $\uparrow$ $Bu(t_2)d\tau$



$\vdots$

∞

$$\underbrace{x(t) = e^{At}x_0 + \int_0^t e^{A(t-\tau)}Bu(\tau)\,d\tau}$$

$\downarrow$ convolution integral

$$e^{At} * Bu(t)$$

e.g. $\dot{x} = -2x + u$

$x_0 = 10$

$u = \sin(t)$

$$\therefore x(t) = 10e^{-2t} + \int_0^t e^{-2(t-\tau)}\sin(\tau)\,d\tau$$

In Matlab

$\dot{x} = Ax + Bu$ u = inputs

$y = Cx + Du$ y = outputs

x = internal states

$\ddot{\theta} = -\sin\theta$

$$\frac{d}{dt}\begin{bmatrix}\theta \\ \omega\end{bmatrix} = \begin{bmatrix}0 & 1 \\ -1 & 0\end{bmatrix}\begin{bmatrix}\theta \\ \omega\end{bmatrix} + \begin{bmatrix}0 \\ 1\end{bmatrix}\tau$$

$$C = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix} \qquad D = \begin{bmatrix}0 \\ 0\end{bmatrix}$$

```
>> A=[0 1; -1 -.1];
>> B=[0;1];
>> C= eye(2);
>> D=[0;0];
>> sys = ss(A,B,C,D);
>> impulse(sys, 100);
>> t=0:0.01:50;
>> u=0*t;
>> u(1001:2000) = (1:1000)/10000;
>> u(2001:3000)=(1000-(1:1000))/10000;
>> plot(t,u)
>> lsim(sys,u,t)
```