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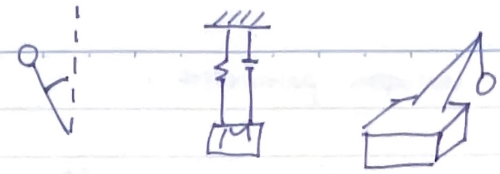
ME564 L12

$\dot{x} = Ax + f(t)$

$\dot{x} = f(x) + g(t)$



Differential Eqns w/ forcing

(*) $\ddot{x} + 3\dot{x} + 2x = 0$ homogeneous

(**) $\ddot{x} + 3\dot{x} + 2x = \underbrace{e^{-3t}}_{\text{forcing term}}$ inhomogeneous

Part I : solve (*) to find homogeneous solution

$x = e^{\lambda t}$
$\dot{x} = \lambda e^{\lambda t}$
$\ddot{x} = \lambda^2 e^{\lambda t}$

$[\lambda^2 + 3\lambda + 2] e^{\lambda t} = 0$

$(\lambda + 2)(\lambda + 1) = 0$

$\lambda = -1, -2$

$\boxed{x(t) = k_1 e^{-t} + k_2 e^{-2t}}$

Part 2: Solve ** to find particular solution

$x_p(t) = k e^{-3t}$ (guessing will not work in general)

as $e^{\lambda t}$, $\cos t$, $\sin t$ are solutions of differential equations, we will be able to let $x_p(t) = k e^{-3t}$

$x_p(t) = k e^{-3t}$
$\dot{x}_p(t) = -3k e^{-3t}$
$\ddot{x}_p(t) = 9k e^{-3t}$

$(9k - 9k + 2k) e^{-3t} = e^{-3t}$

$2k e^{-3t} = e^{-3t}$

$k = \frac{1}{2}$

$\boxed{x_p = \frac{1}{2} e^{-3k}}$

method of undetermined coefficient

Part 3 :

$\boxed{x(t) = \underbrace{k_1 e^{-t} + k_2 e^{-2t}}_{\text{initial condition}} + \underbrace{\frac{1}{2} e^{-3t}}_{\text{forcing.}}}$

ODE is linear (superposition

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method of variation parameters.

$\ddot{x}+3\dot{x}+2x=0 \rightarrow \ddot{x}+3\dot{x}+2x=e^{-3t}$

$x(t)= k_1 e^{-t}+k_2 e^{-2t}$

$\Downarrow$

$x(t)= u_1(t)e^{-t}+u_2(t)e^{-2t}$ (assumed)

$\dot{x}(t)= -u_1(t)e^{-t} - 2u_2(t)e^{-2t}+ \underbrace{\dot{u}_1(t)e^{-t} + \dot{u}_2(t)e^{-2t}}$

$\hookrightarrow$ let them $=0$

constraint 1.

$\ddot{x}(t)= -\dot{u}_1 e^{-t}+u_1 e^{-t} - 2\dot{u}_2 e^{-2t}+ 4u_2 e^{-2t}$

$\ddot{x}+3\dot{x}+2x$

$\Rightarrow \underbrace{[u_1-3u_1+2u_1]}_{=0}e^{-t}+ \underbrace{[4u_2-6u_2+2u_2]}_{=0}e^{-2t} - \dot{u}_1 e^{-t} - 2\dot{u}_2 e^{-2t} = e^{-3t}$

Constraint 2: $-\dot{u}_1 e^{-t}-2\dot{u}_2 e^{-2t}= e^{-3t}$

$\dot{u}_1 e^{-t}+\dot{u}_2 e^{-2t}=0$

$\dot{u}_1 e^{-t}+2\dot{u}_2 e^{-2t}=-e^{-3t}$

$\dot{u}_1(t)= e^{-2t}$ $\quad u_1(t)= -\frac{1}{2}e^{-2t}+C_1$

$\dot{u}_2(t)= -e^{-t}$ $\quad u_2(t)= e^{-t}+C_2$

$x(t)= (-\frac{1}{2}e^{-2t}+k_1)e^{-t}+ (e^{-t}+k_2)e^{-2t}$

$= \frac{1}{2}e^{-3t}+k_1 e^{-t}+k_2 e^{-2t}$

"Linear" systems :

$\dot{x} = Ax + \underline{\underline{Bu}}$

external forcing

if x_1 is a solution : $\dot{x}_1 = Ax_1$

& if x_2 is a solution: $\dot{x}_2 = Ax_2$

then

$x = k_1 x_1 + k_2 x_2$ is also a solution

$\frac{d}{dt} x = k_1 \dot{x}_1 + k_2 \dot{x}_2$

$Ax = A[k_1 x_1 + k_2 x_2] = \underset{\dot{x}_1}{\underset{\parallel}{\underline{k_1 A x_1}}} + \underset{\dot{x}_2}{\underset{\parallel}{\underline{k_2 A x_2}}}$

$= k_1 \dot{x}_1 + k_2 \dot{x}_2$

$\therefore$ $k_1 x_1 + k_2 x_2$ is a soln.