ME564 L11

$\dot{x} = Ax$

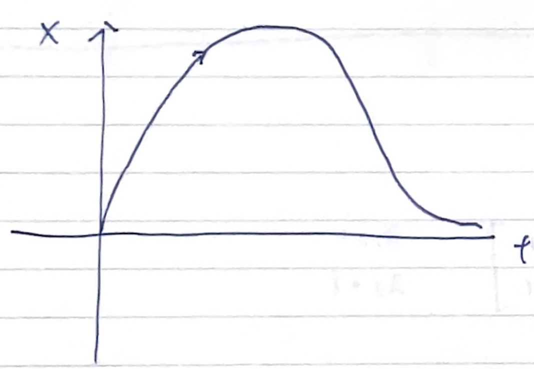
Nearly degenerate systems A ...

... e-vecs of A are nearly parallel

$A = \begin{bmatrix} -0.009 & 1 \\ 0 & -0.01 \end{bmatrix}$ $\quad \left.\begin{matrix} \lambda_1 = -0.01 \\ \lambda_2 = -0.009 \end{matrix}\right\}$ stable

$\xi_1 : [A - \lambda_1 I]\xi_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0.001 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \xi_1 = \begin{bmatrix} 1 \\ -0.001 \end{bmatrix}$

$\xi_2 : [A - \lambda_2 I]\xi_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 \\ 0 & -0.001 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \xi_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$



grow first in a transient fashion
then damped it out.

Jordan canonical form

$\dot{x} = Ax \quad A = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$ when stable λ

$x(t) = e^{At}x(0)$

$A = \underbrace{\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}}_{S} + \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{T}$

$e^{s+t} = e^s e^t$

$e^{S+T} \neq e^S e^T$ generally

yet

$e^{S+T} = e^S e^T$ if $ST = TS$

$ST = \begin{bmatrix} 0 & \lambda \\ 0 & 0 \end{bmatrix}$

$TS = \begin{bmatrix} 0 & \lambda \\ 0 & 0 \end{bmatrix}$

$ST = TS$

$\therefore e^{At} = e^{St}e^{Tt} \quad e^{Tt} = I + Tt + \underbrace{\tfrac{1}{2}T^2t^2 + \dots}_{0} = I + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}t = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$

(proof it via $(S+T)^n = n! \sum_{j+k=n} \frac{S^j T^k}{j!\,k!}$ binomial expansion)

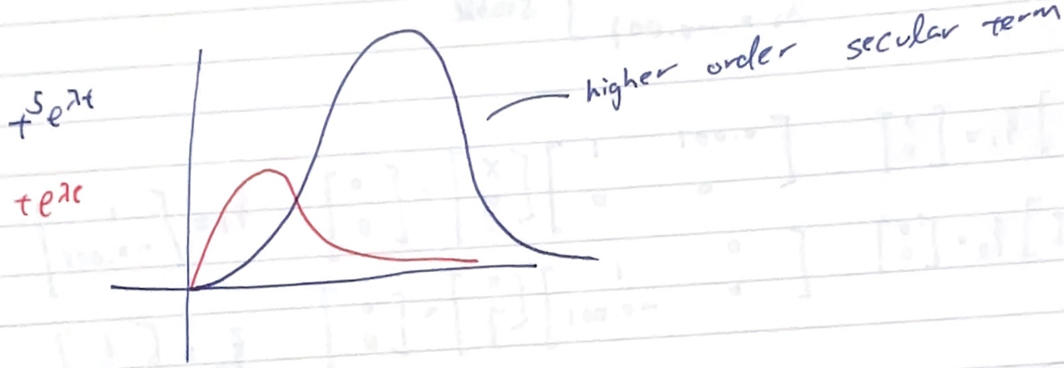
$= \begin{bmatrix} e^{\lambda t} & 0 \\ 0 & e^{\lambda t} \end{bmatrix}\begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} e^{\lambda t} & te^{\lambda t} \\ 0 & e^{\lambda t} \end{bmatrix}$ ※

new term
secular term

No.

Date

$$A = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix} \rightarrow e^{At} = \begin{bmatrix} e^{\lambda t} & te^{\lambda t} & \frac{1}{2}t^2 e^{\lambda t} \\ 0 & e^{\lambda t} & te^{\lambda t} \\ 0 & 0 & e^{\lambda t} \end{bmatrix}$$



A tale of two "A" matrices :

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad \lambda_1 = 1$, $\lambda_2 = 1$

$e^{At} = \begin{bmatrix} e^t & 0 \\ 0 & e^t \end{bmatrix}$

$\boxed{A - \lambda I}\, \xi = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\xi_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\xi_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

look @ the

$\text{rank}(A - \lambda I) = 0$

$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ $\quad \lambda_1 = 1$, $\lambda_2 = 1$

$e^{At} = \begin{bmatrix} e^t & te^t \\ 0 & e^t \end{bmatrix}$

$[A - \lambda I]\, \xi = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\quad \xi_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

look @ the

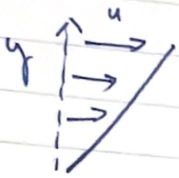
$\text{rank}(A - \lambda I) = 1$

to find more generalized e-vecs

$[A - \lambda I]^2 \xi_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ is non-normal

$A^T A \neq A A^T$



$\dfrac{\partial u}{\partial y} = k$