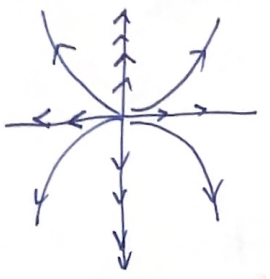


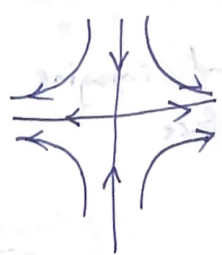
Overview of $2 \times 2 \dot{x} = Ax$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$



unstable source

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



saddle

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\lambda = \pm i$$



neutrally stable

mixed of saddle & unstable.

$$A = \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix}$$

$$\lambda = -1 \pm i$$

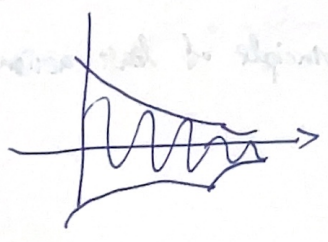
Real(λ) < 0
 \rightarrow stable



stable spiral

$$e^{\lambda t} = e^{(-1 \pm i)t} = e^{-t} e^{\pm it}$$

decay \sin/cos



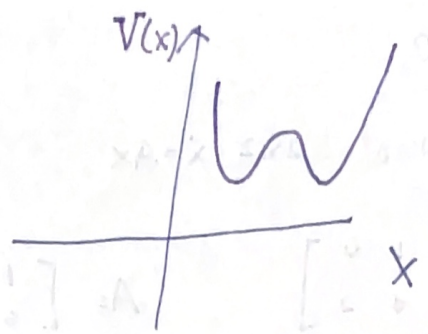
$$A = \begin{bmatrix} -0.1 & 0 \\ 1 & -0.11 \end{bmatrix}$$

eigen values that are really close together.

e-vec close to parallel.



Particle in a potential well
& linearizing nonlinear ODEs



Consider a potential $V(x)$ and imagine sliding a 'bead' on this surface

Force \Rightarrow Force is $F = -\frac{\partial V}{\partial x}$ $m \ddot{x} = -\frac{\partial U}{\partial x}$ $m=1$

$$\ddot{x} = -\frac{\partial U}{\partial x}$$

Let's write Lagrange

kinetic energy $T = \frac{1}{2} \dot{x}^2$

pot. $U(x)$

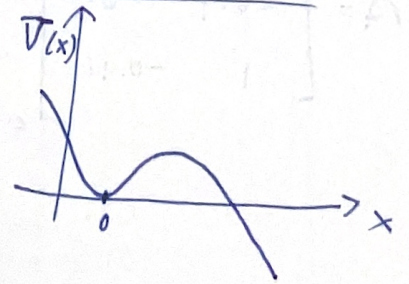
Lagrangian: $L(x, \dot{x}) = T(\dot{x}) - U(x)$

Euler Lagrangian Equation: $\frac{d}{dt} \frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0$

principle of least action

$$\ddot{x} = \frac{\partial L}{\partial x} = -\frac{\partial U}{\partial x}$$

Ex: $\ddot{x} = -x + x^2$ (i.e. $U(x) = \frac{x^2}{2} - \frac{x^3}{3}$)



\downarrow
 $\dot{x} = v$
 $\dot{v} = -x + x^2$ } fixed points $\rightarrow x=0, 1$
 $v=0$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\lambda_1 [1] \quad \lambda_2 [-1]$$

$$\frac{Df}{D[x]} = \begin{bmatrix} 0 & 1 \\ -1+2x & 0 \end{bmatrix};$$

FP1 $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\frac{Df}{D[x]} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

FP2 $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

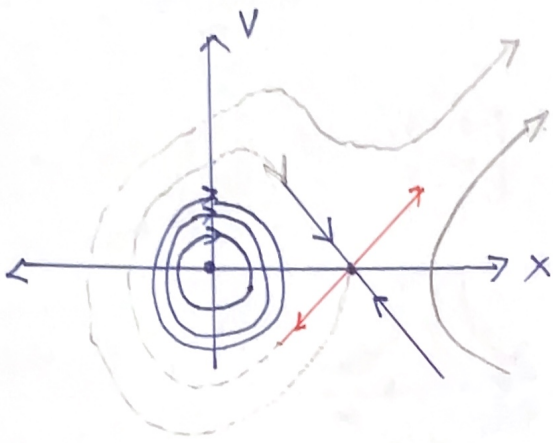
$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

center

unstable

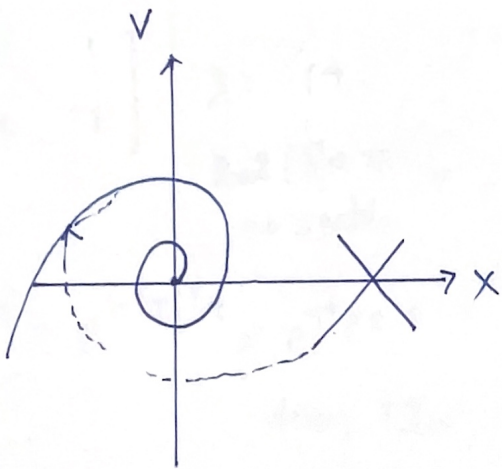
$$\lambda = 1/1$$





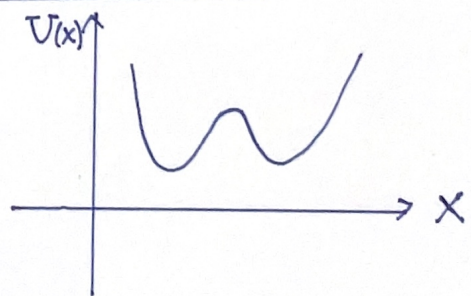
if add damping

$$\ddot{x} = -x + x^2 - \dot{x}$$



$$\ddot{x} = -x^3 + x - \dot{x}$$

optional damping



$$\dot{x} = y$$

$$\dot{y} = x - x^3 - y$$

FPS:

$$y = 0$$

$$x = 0 \pm 1$$

$$-\frac{1}{4}x^4 + \frac{1}{2}x^2 +$$

$$\frac{Df}{Dx} = \begin{bmatrix} 0 & 1 \\ 1-3x^2 & 0 \end{bmatrix}$$

FPS:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\frac{Df}{Dx} = \begin{bmatrix} 0 & 1 \\ 1-3x^2 & -1 \end{bmatrix}$$

saddle ± 1

center $\pm i$

