- Recall

$f(x, W) = Wx + b$ → 

$x$: image vector; $W$: weight

- TODO
  - define a loss function
  - algorithm to do optimization

- dataset

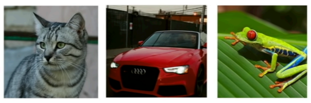
$\{(x_i, y_i)\}_{i=1}^N$ — $x_i$ is image, $y_i$ is label

- cost function

$L = \frac{1}{N} \sum_i L_i(f(x_i, W), y_i)$

- multi-class SVM

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



| | | | |
|---|---|---|---|
| cat | **3.2** | 1.3 | 2.2 |
| car | 5.1 | **4.9** | 2.5 |
| frog | -1.7 | 2.0 | **-3.1** |

**Multiclass SVM loss:**

Given an example $(x_i, y_i)$ where $x_i$ is the image and where $y_i$ is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

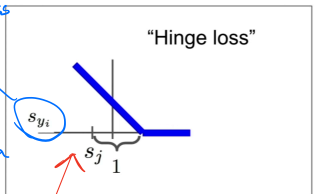
the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \begin{cases} 0 & \text{if } s_{y_i} \geq s_j + 1 \\ s_j - s_{y_i} + 1 & \text{otherwise} \end{cases} = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

**Multiclass SVM loss:**

score of the true class

we are happy if the true class is higher than others

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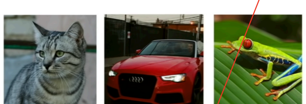

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|---|---|---|---|
| cat | **3.2** | 1.3 | 2.2 |
| car | 5.1 | **4.9** | 2.5 |
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| Losses: | 2.9 | | |

**Multiclass SVM loss:**

Given an example $(x_i, y_i)$ where $x_i$ is the image and where $y_i$ is the (integer) label,

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the SVM loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

= max(0, 5.1 - 3.2 + 1)
+max(0, -1.7 - 3.2 + 1)
= max(0, 2.9) + max(0, -3.9)
= 2.9 + 0
= 2.9

$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$
$= \max(0, 1.3 - 4.9 + 1) + \max(0, 2.0 - 4.9 + 1)$
$= 0$

$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$
$= \max(0, 2.2 - -3.1 + 1)$
$= \max(0, 2.5 - -3.1 + 1)$
$= 12.9$

$\frac{1}{3}(2.9 + 12.9) = 5.27$

→ W not good
continue to optimize!

- is optimal W unique?
  A: No, 2W is also optimal

Qs

- What's gonna happen is score for car is changed a bit?
  A: loss will not change. will still be 0 for car total loss still the same.
- min/max for loss?
  A: min = 0
  max = ∞
- @ init. W, W is small, all $s \approx 0$ what is the loss?
  A: no. classes − 1
- what if the sum includes correct class:
  A: loss + 1
- if we use sum instead of mean?
  A: Questions doesn't change
- what if we use $L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)^2$
  A: algo different

- Regularization

$L(W) = \underbrace{\frac{1}{N} \sum_{i=1}^N L_i(f(x_i, W), y_i)}_{\text{Data Loss}} + \lambda \underbrace{R(W)}_{\text{Regularization: Model should be simple so it works on data}}$

$\lambda$: trade-off hyperparameter

## Regularization

$\lambda$ = regularization strength (hyperparameter)

$$L = \frac{1}{N} \sum_{i=1}^N \sum_{j \neq y_i} \max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + 1) + \boxed{\lambda R(W)}$$

In common use:

**L2 regularization** $R(W) = \sum_k \sum_l W_{k,l}^2$
L1 regularization $R(W) = \sum_k \sum_l |W_{k,l}|$
Elastic net (L1 + L2) $R(W) = \sum_k \sum_l \beta W_{k,l}^2 + |W_{k,l}|$
Max norm regularization (might see later)
Dropout (will see later)
Fancier: Batch normalization, stochastic depth

## L2 Regularization (Weight Decay)

$$x = [1, 1, 1, 1] \qquad R(W) = \sum_k \sum_l W_{k,l}^2$$

$$w_1 = [1, 0, 0, 0]$$

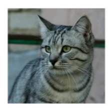
$$w_2 = [0.25, 0.25, 0.25, 0.25]$$

(If you are a Bayesian: L2 regularization also corresponds MAP inference using a Gaussian prior on W)

$$w_1^T x = w_2^T x = 1$$

- softmax classifier (multinomial logistic regression)

**Softmax Classifier** (Multinomial Logistic Regression)



**scores = unnormalized log probabilities of the classes.**

$P(Y = k | X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$ where $s = f(x_i; W)$

softmax

Want to maximize the log likelihood, or (for a loss function) to minimize the negative log likelihood of the correct class:

$$\boxed{L_i = -\log P(Y = y_i | X = x_i)}$$

$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$

| | |
|---|---|
| cat | **3.2** |
| car | 5.1 |
| frog | -1.7 |

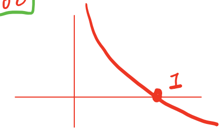
↓ exp

24.5
164.0
0.18

→ normalized

0.13
0.87
0.00

$L_i = -\log(0.13) = 0.89$



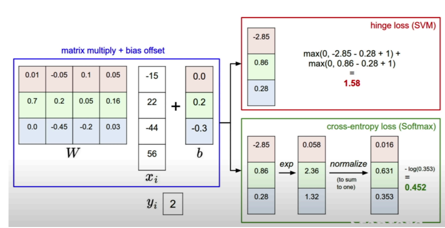

Q: min/max of "loss"

A: ⇒ 0 ∞
practical, will $L_i = 0$ is hard to get as we need

$\frac{e^{s_{y_i}}}{\sum_j e^{s_j}} = 1$

and need $s_{y_i} \to \infty$

∴ will never get to 0



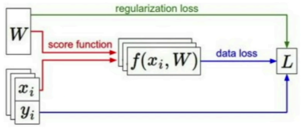
# Recap

- We have some dataset of (x,y)
- We have a **score function:** $s = f(x; W) \overset{\text{e.g.}}{=} Wx$
- We have a **loss function**:

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$ Softmax

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$ SVM

$$L = \frac{1}{N}\sum_{i=1}^{N} L_i + \underbrace{R(W)}_{\text{regularization}}$$ Full loss




- How do we do it? Optimization
  - Strategy: Random search
  - Follow the slope: Gradient Descent

- 

```
# Vanilla Gradient Descent

while True:
  weights_grad = evaluate_gradient(loss_fun, data, weights)
  weights += - step_size * weights_grad # perform parameter update
```

hyperparameter (learning rate) — step

monte-carlo estimate of some expectation

## Stochastic Gradient Descent (SGD)

$$L(W) = \frac{1}{N}\sum_{i=1}^{N} L_i(x_i, y_i, W) + \lambda R(W)$$

$$\nabla_W L(W) = \frac{1}{N}\sum_{i=1}^{N} \nabla_W L_i(x_i, y_i, W) + \lambda \nabla_W R(W)$$

Full sum expensive when N is large!

Approximate sum using a **minibatch** of examples
32 / 64 / 128 common

```
# Vanilla Minibatch Gradient Descent

while True:
  data_batch = sample_training_data(data, 256) # sample 256 examples
  weights_grad = evaluate_gradient(loss_fun, data_batch, weights)
  weights += - step_size * weights_grad # perform parameter update
```

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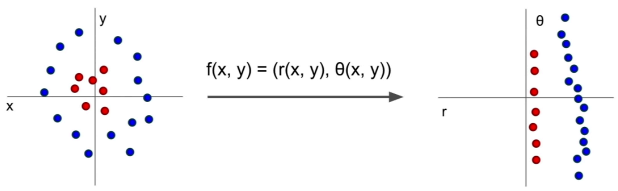
- Image Features (instead of raw data)

## Image Features: Motivation

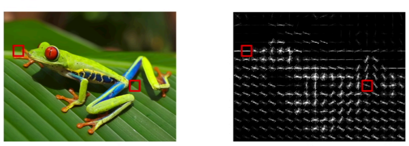



Cannot separate red and blue points with linear classifier

After applying feature transform, points can be separated by linear classifier

- image color histogram
- how about MIMO radar?

Example: Histogram of Oriented Gradients (HoG)



Divide image into 8x8 pixel regions
Within each region quantize edge direction into 9 bins

Example: 320x240 image gets divided into 40x30 bins; in each bin there are 9 numbers so feature vector has 30*40*9 = 10,800 numbers

Example: Bag of Words